

## 4-3 day 4 Connecting the Graphs of $f$ , $f'$ , & $f''$

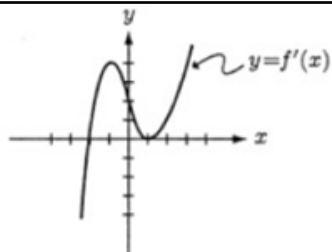
### Learning Objectives:

I can sketch a graph of the derivative of a function from the graph of the function.

I can sketch a graph of the function given the graph of the derivative of a function and a point on the curve.

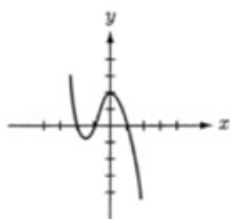
I can identify extrema and inflection points of a function from the graph of the derivative (or second derivative) of a function.

I can identify when a function is increasing, decreasing, concave up, or concave down from the graph of the derivative (or second derivative) of a function.

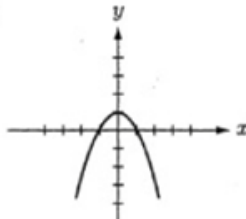


The graph of the derivative of  $f$  is shown in the figure above. Which of the following could be the graph of  $f$ ?

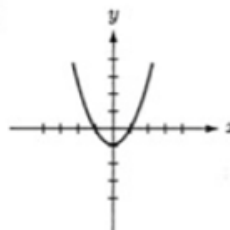
(A)



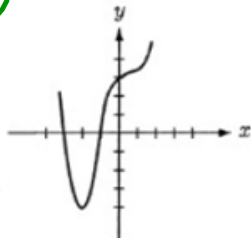
(B)



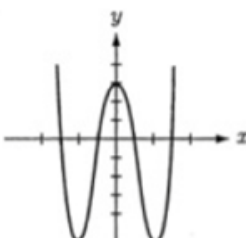
(C)



(D)



(E)



1. (NC) If  $g'(x) = \ln(x-2)$ , then the graph of  $y=f(x)$  is decreasing if and only if

(a)  $2 < x < 3$

(b)  $0 < x$

(c)  $0 < x < 1$

(d)  $x > 1$

(e)  $x > 2$

2. (NC) For  $x \neq 0$ , the slope of the tangent to  $y = x \cos x$  equals 0 whenever

(a)  $\tan x = -x$

(b)  $\tan x = 1/x$

(c)  $\tan x = x$

(d)  $\sin x = x$

(e)  $\cos x = x$

3. (NC) A relative maximum of the function  $f(x) = \frac{(\ln x)^2}{x}$  occurs at  
(a) 0            (b) 1            (c) 2            (d) e            (e)  $e^2$

4. (NC) An equation of the line tangent to the graph of  $y = x^3 + 3x^2 +$   
at its point of inflection is

- (a)  $y = -3x+1$             (b)  $y=-3x-7$             (c)  $y=x+5$   
(d)  $y = 3x+1$             (e)  $y=3x+7$

Let  $f$  be a function that is even and continuous on the closed interval  $[-3,3]$ . The function  $f$  and its derivative have the properties indicated in the table below.

$x$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	positive	0	negative	-1	negative
$f'(x)$	undefined	negative	0	negative	undefined	positive
$f''(x)$	undefined	positive	0	negative	undefined	negative

- Find the  $x$ -coordinate of each point at which  $f$  attains an absolute maximum value or an absolute minimum value. For each  $x$ -coordinate you give, state whether  $f$  attains an absolute maximum or an absolute minimum.
- Find the  $x$ -coordinate of each point of inflection on the graph of  $f$ . Justify your answer.
- Sketch a graph of a function with all of the given characteristics of  $f$ .

Let  $f$  be a function that is continuous on the interval  $[-1,4]$  with the properties given in the table below.

$x$	-1	$-1 < x < 0$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 4$	4
$f(x)$	1	positive	0	negative	-1	negative	0	positive	3
$f'(x)$	-6	negative	-1	negative	DNE	positive	<del>x</del> 1	positive	8
$f''(x)$	3	positive	0	negative	DNE	negative	0	positive	4

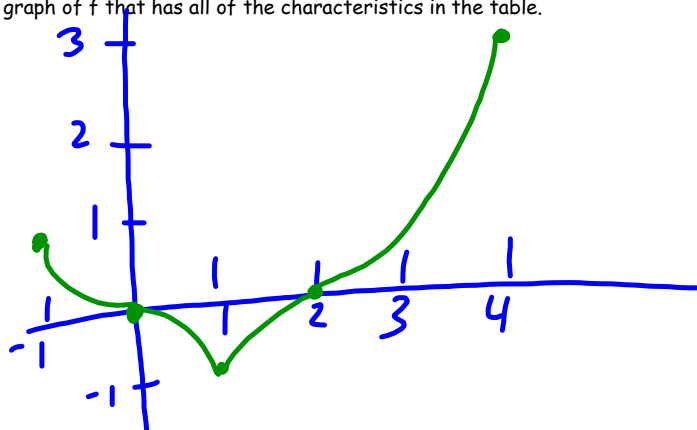
a. Find all values of  $x$  at which  $f$  has extrema. Determine whether  $f$  has a relative maximum or minimum. Justify your answer.

b. Find the maximum value of  $f$ .   
 $x = 4, x = -1,$   
 $x = 1$   $f'$  switches from  $-$  to  $+$   
 $\text{max}$   
 $3$   
 $\infty \text{ min}$

c. Find any inflection points.

$x = 0, x = 2$

d. Sketch a graph of  $f$  that has all of the characteristics in the table.



# Homework

pg 215 # 32, 41-47, 49, 50,  
60